SOLUTIONS TO SELECTED QUESTIONS IN HOMEWORK 7

MATH 241

19.1.4

Proof. $\lim |(1+i)^n| = 2^{\frac{n}{2}}$ goes to infinity, so the sequence does not converge. □

19.1.10

Proof. $\tan\frac{\pi}{2} = \infty$, so $\lim \tan^{-1} n = \frac{\pi}{2}$. Therefore $\lim(e^{\frac{1}{n}} + 2(\tan^{-1} n)i) = 1 + \pi i$. □

19.1.13

Proof. The partial sums $S_n = (\frac{1}{1+2i} - \frac{1}{2+2i}) + (\frac{1}{2+2i} - \frac{1}{3+2i}) + \cdots + (\frac{1}{n+2i} - \frac{1}{n+1+2i}) = \frac{1}{1+2i} - \frac{1}{n+1+2i}$ converges $\frac{1}{1+2i}$. So the series converges to $\frac{1}{1+2i}$. □

19.1.29

Proof. $\lim(\frac{1}{k2^k})^{\frac{1}{k}} = \lim \frac{1}{2k^{\frac{1}{k}}} = \frac{1}{2}$, so the radius of limit is $R = 2$. On the circle of convergence, $|\frac{(z-i)^k}{k2^k}| = \frac{1}{k}$, but $\sum \frac{1}{k}$ diverges. So the power series $\sum \frac{(z-i)^k}{k2^k}$ is not absolutely convergent on the circle of convergence. When $z = i - 1$, $\sum \frac{(-1)^k}{k2^k}$ converges by the alternating criterion. □

Spring 08, #5(b)

Proof. $\lim \frac{n-i}{n+i} = 1$ is not zero, so the series cannot converge. □

Spring 08, #6(a)

Proof. $\lim(n^4)^{\frac{1}{n}} = \lim(n^{\frac{1}{n}})^4 = 1$, so the radius of convergence is $\frac{1}{1} = 1$. □

Spring 08, #6(b)

Proof. $\lim \frac{\frac{1}{((n+1)!)^2}}{\frac{1}{(n!)^2}} = \lim \frac{(n!)^2}{((n+1)!)^2} = \lim \frac{1}{(n+1)^2} = 0$, so the radius of convergence is $\frac{1}{0} = \infty$. □